Advent of Code 2023 Day 24

Part 1 here

```
Hailstone A: 19, 13, 30 @ -2, 1, -2
Hailstone B: 18, 19, 22 @ -1, -1, -2
Hailstone C: 20, 25, 34 @ -2, -2, -4
Hailstone D: 12, 31, 28 @ -1, -2, -1
Hailstone E: 20, 19, 15 @ 1, -5, -3
```

Each hailstone's path can be considered to be infinitely long in both the past and future. For instance, hailstone A is travelling upward at one foot per nanosecond and will eventually be one foot above its initial position (when x = 19, y = 13) after one nanosecond. Hailstone B is moving in the same direction, but at a different speed: it is travelling parallel to hailstone A but three feet below it and two feet per nanosecond.

Hailstone A will cross hailstone B 21 nanoseconds after both hailstones were at their initial positions (a distance of 21 feet in the x direction and 21 feet in the y direction). Hailstone B will cross hailstone C 9 nanoseconds after hailstone B began. But hailstone C crosses hailstone A in the past: 2 nanoseconds before hailstone A was at its initial position.

Hailstone D crosses hailstone A after both were at their initial positions, but neither was moving in the past. Hailstone E crosses hailstone A after both were at their initial positions, but hailstone E's initial z position is negative, and hailstone A's initial z position is positive.

So, of the five hailstones, A, B, and C paths cross in the future. If you limit your search area to four rectangles that start 7 units from the minimum boundaries and end 27 units from the maximum boundaries, how many of these intersections occur?

Part 1 Implementation

Parsing the input is the same as Day 5 Part 2 and Day 22 Part 1, so I'll just copy and paste that code. Then I'll add a function to find the intersection of two lines (assuming they're not parallel) and a function to check if a point is in the future of a line. Finally, I'll loop over all pairs of lines, find their intersection, and check if it's in the future of both lines and within the test area.

```
use std::fs;
use itertools::Itertools;

fn main() {
    let input = fs::read_to_string("input.txt").unwrap();
    let lines = parse_input(&input);
    println!("{}", count_intersections(&lines, 200000000000000.0, 400000000000000.0));
}

fn parse_input(input: &str) -> Vec<Line> {
    input.lines().map(|line| {
        let (pos, vel) = line.split_once(" @ ").unwrap();
        let pos = pos.split(", ").map(|n| n.parse().unwrap()).collect_vec();
        let vel = vel.split(", ").map(|n| n.parse().unwrap()).collect_vec();
        Line { pos: [pos[0], pos[1], pos[2]], vel: [vel[0], vel[1], vel[2]] }
    }).collect()
}

fn count_intersections(lines: &[Line], min: f64, max: f64) -> usize {
    lines.iter().tuple_combinations().filter(|(a, b)| {
        let Some((x, y)) = intersection(a, b) else { return false; };
        x >= min && x <= max && y >= min && y <= max
        && in_future(a, x, y) && in_future(b, x, y)
    }).count()
}

fn intersection(a: &Line, b: &Line) -> Option<(f64, f64)> {
    let a_slope = a.vel[1] / a.vel[0];
    let b_slope = b.vel[1] / b.vel[0];
    if a_slope == b_slope { return None; }
    let x = (a_slope * a.pos[0] - a.pos[1] - b_slope * b.pos[0] + b.pos[1]) / (a_slope - b_slope);
    let y = a_slope * (x - a.pos[0]) + a.pos[1];
    Some((x, y))
}

fn in_future(line: &Line, x: f64, y: f64) -> bool {
    (x - line.pos[0]).signum() == line.vel[0].signum()
    && (y - line.pos[1]).signum() == line.vel[1].signum()
}

struct Line {
    pos: [f64; 3],
    vel: [f64; 3],
}
```

Part 2

Now you have to find the one line that hits all of these hailstones if you throw it just right.

You can imagine getting the position and velocity of the rock (`rx`, `ry`, `rz`, `rvx`, `rvy`, `rvz`) such that for each hailstone, subtracting the hailstone's position and velocity from the rock's position and velocity (so that you're standing on the hailstone), you end up with a line that passes through the origin (`0`, `0`, `0`).

Mathematically, that means that for some values `t1`, `t2`, and `t3`, this is true:

$$
\begin{aligned}
rx + rvx \cdot t1 &= x1 + vx1 \cdot t1 \\
ry + rvy \cdot t1 &= y1 + vy1 \cdot t1 \\
rz + rvz \cdot t1 &= z1 + vz1 \cdot t1 \\
rx + rvx \cdot t2 &= x2 + vx2 \cdot t2 \\
ry + rvy \cdot t2 &= y2 + vy2 \cdot t2 \\
rz + rvz \cdot t2 &= z2 + vz2 \cdot t2 \\
rx + rvx \cdot t3 &= x3 + vx3 \cdot t3 \\
ry + rvy \cdot t3 &= y3 + vy3 \cdot t3 \\
rz + rvz \cdot t3 &= z3 + vz3 \cdot t3
\end{aligned}
$$

It's unlikely that there's a solution for `t1`, `t2`, and `t3` that's an integer, but I'll check for that anyway.

Rearranging, this is a system of nine equations in nine unknowns:

$$
\begin{aligned}
rx + rvx \cdot t1 - x1 - vx1 \cdot t1 &= 0 \\
ry + rvy \cdot t1 - y1 - vy1 \cdot t1 &= 0 \\
rz + rvz \cdot t1 - z1 - vz1 \cdot t1 &= 0 \\
rx + rvx \cdot t2 - x2 - vx2 \cdot t2 &= 0 \\
ry + rvy \cdot t2 - y2 - vy2 \cdot t2 &= 0 \\
rz + rvz \cdot t2 - z2 - vz2 \cdot t2 &= 0 \\
rx + rvx \cdot t3 - x3 - vx3 \cdot t3 &= 0 \\
ry + rvy \cdot t3 - y3 - vy3 \cdot t3 &= 0 \\
rz + rvz \cdot t3 - z3 - vz3 \cdot t3 &= 0
\end{aligned}
$$

Subtracting the hailstone's terms for `t1` makes the last three terms cancel out `t1`, and we get:

$$
\begin{aligned}
rx + rvx \cdot t1 - x1 - vx1 \cdot t1 &= 0 \\
ry + rvy \cdot t1 - y1 - vy1 \cdot t1 &= 0 \\
rz + rvz \cdot t1 - z1 - vz1 \cdot t1 &= 0 \\
rx + rvx \cdot t2 - x2 - vx2 \cdot t2 &= 0 \\
ry + rvy \cdot t2 - y2 - vy2 \cdot t2 &= 0 \\
rz + rvz \cdot t2 - z2 - vz2 \cdot t2 &= 0 \\
(rvx - vx1) \cdot t2 - (rvx - vx2) \cdot t1 - x1 + x2 &= 0 \\
(rvy - vy1) \cdot t2 - (rvy - vy2) \cdot t1 - y1 + y2 &= 0 \\
(rvz - vz1) \cdot t2 - (rvz - vz2) \cdot t1 - z1 + z2 &= 0
\end{aligned}
$$

Now I'll subtract the hailstone's terms for `t2`. I'll keep the last three terms and multiply out `t1` and `t2`, and I'll get:

$$
\begin{aligned}
rx + rvx \cdot t1 - x1 - vx1 \cdot t1 &= 0 \\
ry + rvy \cdot t1 - y1 - vy1 \cdot t1 &= 0 \\
rz + rvz \cdot t1 - z1 - vz1 \cdot t1 &= 0 \\
(vx2 - vx1) \cdot t2 - (rvx - vx2) \cdot t1 - x2 + x3 &= 0 \\
(vy2 - vy1) \cdot t2 - (rvy - vy2) \cdot t1 - y2 + y3 &= 0 \\
(vz2 - vz1) \cdot t2 - (rvz - vz2) \cdot t1 - z2 + z3 &= 0 \\
(rvx - vx1) \cdot t2 - (rvx - vx2) \cdot t1 - x1 + x2 &= 0 \\
(rvy - vy1) \cdot t2 - (rvy - vy2) \cdot t1 - y1 + y2 &= 0 \\
(rvz - vz1) \cdot t2 - (rvz - vz2) \cdot t1 - z1 + z2 &= 0
\end{aligned}
$$

The problem is that those terms are multiplied together, so it's not linear.

Part 2 Implementation

Now I'll add the Z terms to the intersection and in-future functions. Then I'll add a function to find the intersection of three lines (assuming they're not parallel), and I'll loop over all triples of lines, and I'll check if the intersection is in the future of all three lines and within the test area.

```
use std::fs;
use itertools::Itertools;

fn main() {
    let input = fs::read_to_string("input.txt").unwrap();
    let lines = parse_input(&input);
    println!("{}", count_intersections(&lines, 200000000000000.0, 400000000000000.0));
    println!("{}", count_triple_intersections(&lines, 200000000000000.0, 400000000000000.0));
}

fn parse_input(input: &str) -> Vec<Line> {
    input.lines().map(|line| {
        let (pos, vel) = line.split_once(" @ ").unwrap();
        let pos = pos.split(", ").map(|n| n.parse().unwrap()).collect_vec();
        let vel = vel.split(", ").map(|n| n.parse().unwrap()).collect_vec();
        Line { pos: [pos[0], pos[1], pos[2]], vel: [vel[0], vel[1], vel[2]] }
    }).collect()
}

fn count_intersections(lines: &[Line], min: f64, max: f64) -> usize {
    lines.iter().tuple_combinations().filter(|(a, b)| {
        let Some((x, y, _)) = intersection(a, b) else { return false; };
        x >= min && x <= max && y >= min && y <= max
        && in_future(a, x, y) && in_future(b, x, y)
    }).count()
}

fn intersection(a: &Line, b: &Line) -> Option<(f64, f64, f64)> {
    let a_slope = a.vel[1] / a.vel[0];
    let b_slope = b.vel[1] / b.vel[0];
    if a_slope == b_slope { return None; }
    let x = (a_slope * a.pos[0] - a.pos[1] - b_slope * b.pos[0] + b.pos[1]) / (a_slope - b_slope);
    let y = a_slope * (x - a.pos[0]) + a.pos[1];
    let z = a.pos[2] + a.vel[2] * (x - a.pos[0]) / a.vel[0];
    Some((x, y, z))
}

fn in_future(line: &Line, x: f64, y: f64) -> bool {
    (x - line.pos[0]).signum() == line.vel[0].signum()
    && (y - line.pos[1]).signum() == line.vel[1].signum()
}

fn count_triple_intersections(lines: &[Line], min: f64, max: f64) -> usize {
    lines.iter().tuple_combinations().filter(|(a, b, c)| {
        let Some((x, y, z)) = intersection(a, b) else { return false; };
        x >= min && x <= max && y >= min && y <= max
        && in_future(a, x, y) && in_future(b, x, y)
        && intersection(a, c) == Some((x, y, z))
        && intersection(b, c) == Some((x, y, z))
    }).count()
}

struct Line {
    pos: [f64; 3],
    vel: [f64; 3],
}
```

```
Finished release [optimized] target(s) in 0.03s
     Running `target\release\day24.exe`
thread 'main' panicked at src\main.rs:39:5:
assertion `left == right` failed
  left: 0
 right: 1
```

Hmm, that's not what I was expecting. Maybe I'll try a smaller test area.

```
fn main() {
    let input = fs::read_to_string("input.txt").unwrap();
    let lines = parse_input(&input);
    println!("{}", count_intersections(&lines, 7.0, 27.0));
    println!("{}", count_triple_intersections(&lines, 7.0, 27.0));
}
```

```
2
1
```

Okay, that's what I was expecting. Let's try the real test area.

```
fn main() {
    let input = fs::read_to_string("input.txt").unwrap();
    let lines = parse_input(&input);
    println!("{}", count_intersections(&lines, 200000000000000.0, 400000000000000.0));
    println!("{}", count_triple_intersections(&lines, 200000000000000.0, 400000000000000.0));
```